



Modelling Environmental Influences on Wanderer Spiders in the Langhe Region (Piemonte – NW Italy)¹

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Abstract: We develop a mathematical model for understanding the effect of wanderer spiders as biological controllers of the insects infesting vineyards, thus accounting also for the role played by residual wood and green patches as spiders habitat in the otherwise homogeneous landscape of the Langhe region. We then extend the deterministic model allowing random fluctuations around the coexistence equilibrium. The stochastic stability properties of the model are investigated both analytically and numerically.

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1 Background information

The earliest historical evidence of wine production dates from between 6000 and 5000 B.C. Wine making production improved considerably at the time of the ancient Greeks. Only toward the end of the Roman Empire however, fifth century A.D., modern cultivation techniques became common throughout Europe. During the instabilities of the Middle Ages, the Christian monasteries maintained and developed viticultural practices. They indeed owned and tended the best vineyards in Europe. This in view of the fact that these secure and stable environments had the resources and the interest to make the quality of their vines better. At the time, vinum theologium was considered superior to all others. A wide variety of the *Vitis vinifera* grapes has constituted for centuries the bulk of the European vineyards. In the late 19th century however, the plant louse *phylloxera* was accidentally introduced from North America into European vineyards. *Phylloxera* is a North American insect that kills the vine by feeding on its roots. This pest caused the failure of the first plantations of European grapes in the Eastern United States, but was identified only in the late 19th century. Since around 1860, it spread around the world, perhaps traveling on

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resistant American vines. It fast infected *Vitis vinifera* in every country from France to Australia to California, deeply affecting the vineyard agroecosystems and causing relevant economic damages. Throughout Europe and particularly in Italy, the effects went so far that people suffered up to dying for starvation in consequence of the damages inflicted on vineyards by the invasion of *Phylloxera*.

The Native American grapevines include varieties which are instead resistant to the bug, such as *Vitis labrusca*. These grapes however produce wines with a foxy, animal-like taste. French and American researchers finally saved the world's wine industry by grafting *phylloxera-susceptible* European vines onto resistant Eastern American roots. Virtually all wine grapevines in Europe and California are now grafted to rootstocks of Eastern American origin. However no remedy for *phylloxera* is yet available. This insect thus remains a potential danger to any vineyard not planted with grafted rootstock. In fact in 1979 *phylloxera B* overcame the resistance of the dominant rootstock in Northern California vineyards; thousands of acres subsequently were replanted with more resistant rootstocks.

In addition to *phylloxera*, a number of various other diseases and pests affect the *V. vinifera* on the Pacific slopes. We mention for instance black measles, little-leaf, nematodes, red ticks, rabbits, and gophers. In the Eastern United States vineyards are affected by several insects which cause various damages. In particular fruit is damaged in the spring by the grapevine beetle, eating the new buds; by the grape-berry moth, which makes it to color prematurely. Climbing cutworms hide in the ground during the day and feed on the buds at night. Black rot shrivels the fruit and crown rot destroys some kinds of the vines. Also fungal disease as mildew are to be counted for.

Intensive viticulture involves several other operations to produce high quality wines, apart from prophylaxis of healthy vines and treatment of the diseased ones. These must be carried out continuously throughout the year. Even at the early stages of tending a vineyard, appropriate vines must be selected congenial to the soil at hand. The grower must apply his knowledge, skills and industry in order to plant the vines and then attend them by cutting, layering, grafting and fertilizing. These go on until the final gathering of the crop and the proper wine production and aging.

There is a vast literature on the bad long term consequences of the use of pesticides, both on human health and in terms of environmental costs, since the poison acts on both the pests and even with a more marked effect, on their biological controllers. The pests tend to develop resistance to the treatment and the few ones that survive, not being limited by their natural predators, reproduce and generate population explosions [28]. For instance spraying once a month for 6 months the lemon trees with DDT at 0.5% ensures the explosion of their pest population. The biological control is defined as the use of natural predators of specific pests to keep their densities down so as to have an economically acceptable cost of their damages.

The role of spiders as biocontrol agents has been widely neglected in the past in view of the fact that they possess generalist predatory habits. However Riechert and Lockley [27] argued strongly for the importance of the spider assemblage in limiting densities of insect pests in agroecosystems, postulating that spiders exhibit equilibrium point control of insects, even though spiders will not closely track changes in pest densities.

Among the few other authors who have focused on spider communities in vineyards we recall [3, 7, 8, 9, 10, 12, 16, 20, 21]. The main conclusions of these researches can be summarized in two main points. First of all, in spite of their vegetation structure, vineyards can host a very diversified and abundant spider community. Secondly, a well developed herbaceous layer between vine rows can play a very important role in maintaining a spider community.

Further, recent experiments demonstrate that it is not the individual spider species but rather it is the entire spider community that has a substantial impact upon pests in an agroecosystem [26]. One of the main factor influencing the composition of spider communities is the physiognomy or physical structure of the environment [31]. Wise [34] suggested that the way vegetation structure influences spider communities is rather complex and must take into account several variables like

microclimate, prey availability and exchange with adjacent areas. Due to the strong relationship between spiders and vegetation, changes in specific dominance can often be interpreted as result of habitat perturbations. Experiments involving habitat manipulation provide clear evidence that spiders are extremely sensible to vegetation structure and can react quickly and strongly to brief or sudden changes in it. This has been observed mainly in web-builder species, but several studies provide also interesting results on wanderer species, like wolf spiders. The influence of perturbation on spider communities is particularly interesting in the context of agroecosystems. Agricultural land can be seen as a continuous shifting mosaic of land types, full of human induced or natural perturbations. In this paper we propose a mathematical model incorporating the environmental disturbances.

Spider communities are considered a key assemblage for maintenance of stability and equilibrium in an ecosystem controlled by human management practices. It is increasingly recognized that the preservation of biodiversity is only possible through the (re)establishment of a mosaic of suitable habitat patches at the landscape scale [17].

Vineyards can be regarded as distinctive features of a cultural landscape in view of their role as agroecosystems strongly reflecting the heritage of historical relationships between man and the land. Vineyards can be considered as landmarks of one of the most distinctive Italian landscapes, [24]. In Piedmont (NW Italy), as in other countries, this kind of agroecosystem is facing intense transformations, due to two apparently opposite processes: land abandonment and agricultural intensification. The latter results in the progressive removal of small natural landscape elements, thus enhancing the overall loss of heterogeneity and its consequences on biotic communities, [2, 13, 29]. The Langhe region in Piemonte, is a unique environment characterized by soil texture with prevalent clay loam, an annual precipitation of nearly 900 mm and a high density of vineyards.

The Langhe are a hilly region which can be divided into two portions. The Bassa Langa, a worldwide renowned area for its wines (Barolo, Barbaresco) and truffles (the white truffle of Alba), is situated between the rivers Tanaro on the north and Belbo on the south, with maximal height of about 600 meters on the sea level, with vineyards dominating the landscape and a kind of continental climate. The Alta Langa on the south of the former has higher altitudes, from around 750 to 890 meters on the sea level and colder climate, with a landscape in which more woods alternate to the vineyards.

2 The basic deterministic model

The basic populations which are involved in this model are three, as in [32], while for a different type of model involving wanderer spiders see [33]. First of all we have the insects in open fields and woods, denoted by $f(t)$. We let $v(t)$ denote the population of parasites living on vineyards, which can constitute prey for the spiders. The wanderer spiders population in the whole environment is instead $s(t)$.

The evolution of each population is modelled by the differential equations below. We assume a small carrying capacity W for the wood insects, in view of the reduced extension of their environment with respect to the vineyard, which has instead the larger carrying capacity V , contrary to [32], where Malthus reproduction was considered. The equation for the spiders population dynamics assumes that f and v are the only food source for spiders, so that when the former are absent the latter will die exponentially fast. In all equations predation is accounted for by mass action law terms with suitable signs. Prey are turned into new spiders with “efficiency” constant

$0 < k < 1$. All the parameters are to be considered nonnegative unless otherwise specified.

$$\begin{aligned} \dot{f} &= rf\left(1 - \frac{f}{W}\right) - csf \equiv F(f, s, v) \\ \dot{s} &= s(-a + kbv + kcf) \equiv S(f, s, v) \\ \dot{v} &= ev\left(1 - \frac{v}{V}\right) - bsv \equiv P(f, s, v) \end{aligned} \quad (1)$$

2.1 Preliminary results

By setting $X = (f, s, v)^T \in R^3$ and $R(X) = [F(X), S(X), P(X)]^T$, equations (1) can be rewritten as the system

$$\dot{X} = R(X), \quad (2)$$

together with suitable initial conditions $X(0) = X_0 \in R_+^3$. Here $R : R_+^3 \rightarrow R^3$ and $R \in C^\infty(R^3)$. Since it is a homogeneous system then, any solution of (2) satisfies $X(t) \in R_+^3$ for all $t > 0$, i. e. it remains nonnegative, in other words it is biologically feasible. In addition to positive invariance we now show that all the trajectories are bounded.

By setting $\psi(t) = f(t) + s(t) + v(t)$, we can calculate the derivative of ψ along the solution of (1), to find in view of the assumption $k < 1$

$$\begin{aligned} \frac{d\psi(t)}{dt} &= rf\left(1 - \frac{f}{W}\right) - csf + s(-a + kbv + kcf) + ev\left(1 - \frac{v}{V}\right) - bsv \\ &\leq rf\left(1 - \frac{f}{W}\right) - as + ev\left(1 - \frac{v}{V}\right). \end{aligned}$$

Taking now $\eta > 0$ we have

$$\frac{d\psi}{dt} + \eta\psi \leq rf\left(1 - \frac{f}{W}\right) + \eta f - (a - \eta)s + ev\left(1 - \frac{v}{V}\right) + \eta v,$$

from which by choosing $\eta \leq a$, it follows

$$\frac{d\psi}{dt} + \eta\psi \leq \frac{W(r + \eta)^2}{4r} + \frac{V(e + \eta)^2}{4e} \equiv \ell.$$

The intermediate bounds are respectively the maxima of these suitable functions: $\pi_1(f) \equiv rf\left(1 - \frac{f}{W}\right) + \eta f$ and $\pi_2(v) \equiv ve\left(1 - \frac{v}{V}\right) + \eta v$. From the theory of differential inequalities [5], we then obtain

$$0 < \psi(f, s, v) < \frac{\ell}{\eta}(1 - e^{-\eta t}) + \psi(f(0), s(0), v(0))e^{-\eta t}$$

and for $t \rightarrow \infty$, it follows $0 < \psi < \frac{\ell}{\eta}$, hence all solutions of (1) starting in the first orthant are confined in the region $G = \{(f, s, v)^T \in R_+^3 : \psi = \frac{\ell}{\eta} + \theta \text{ for any } \theta > 0\}$ for all $t \geq T_*$, where T_* depends on the initial values $(f(0), s(0), v(0))^T$.

2.2 Equilibrium points and their stability properties

The feasible equilibria of (1) are given first by the origin $E_0 \equiv (0, 0, 0)$ which is unstable, as the eigenvalues are found to be $r, -a, e$. The axial equilibrium points $E_1 \equiv (W, 0, 0)$ and $E_2 \equiv (0, 0, V)$ are both unstable in view of the eigenvalues $-r, -a + ckW, e$ and $r, -a + kbV, -e$ respectively.

There are three further boundary equilibria. The planar equilibrium $E_3 \equiv (0, s_3, v_3) = (0, \frac{e}{b}(1 - \frac{a}{kbV}), \frac{a}{bk})$ is feasible for $a < kbV$. Its eigenvalues are $r - cs_3, \pm ib\sqrt{ks_3v_3}$ giving a stable or unstable center in the s, v plane depending on whether $r < cs_3$. The second planar equilibrium $E_4 \equiv$

$(f_4, s_4, 0) = (\frac{a}{ck}, \frac{r}{c}(1 - \frac{a}{ckW}), 0)$ is feasible for $a < ckW$. The eigenvalues are the roots of the quadratic equation $\Psi(\lambda) \equiv ckW\lambda^2 + ra\lambda + ar(ckW - a) = 0$ and $\lambda_3 = e - bs_4 > 0$. Since $\Psi(0) > 0$ and $\Psi'(0) > 0$ we obtain two roots with negative real parts. Hence E_4 is stable if and only if $c^2kWe < br(ckW - a)$. The last planar equilibrium $E_5 \equiv (f_5, 0, v_5) = (W, 0, V)$ has the eigenvalues $-r, -e, k(bV + cW) - a$ and is then stable for $a > k(bV + cW)$.

The interior equilibrium point $E^* \equiv (f^*, s^*, v^*)$, where

$$f^* = \frac{W(rkb^2V + cae - ckbV)}{k(rb^2V + c^2We)},$$

$$s^* = \frac{er(kbV + kcW - a)}{k(rb^2V + c^2We)},$$

$$v^* = \frac{V(ekc^2W + bra - brkcW)}{k(rb^2V + c^2We)},$$

is feasible for $rkb^2V + ca > ckbV$, $kbV + kcW > a$ and $abr + c^2ekW > ckWbr$. Its characteristic equation is given by $\lambda^3 + Q_1\lambda^2 + Q_2\lambda + Q_3 = 0$, where

$$Q_1 = \frac{eWv^* + rVf^*}{WV},$$

$$Q_2 = \frac{erf^*v^*}{WV} + kb^2s^*v^* + kc^2s^*f^*,$$

$$Q_3 = (\frac{rkb^2}{W} + \frac{ec^2k}{V})kf^*s^*v^*.$$

According to the Routh Hurwitz criterion E^* is locally asymptotically stable if and only if $Q_1 > 0$, $Q_3 > 0$ and $Q_1Q_2 - Q_3 > 0$, conditions that in our case are easily seen to be always satisfied. Thus, when feasible E^* is always stable.

3 The stochastic model

The above discussion rests on the assumption that the environmental parameters involved in the system are all constants irrespective of time and environmental fluctuations. In this Section we will consider the effect of environmental fluctuations on the model and the stochastic stability of the coexisting equilibrium associated with the deterministic model.

There are two ways to develop the stochastic model corresponding to an existing deterministic one. Firstly, one can replace the environmental parameters in the deterministic model by some random parameters, e.g. the growth rate parameter ‘ r ’ can be replaced by $r_0 + \epsilon\gamma(t)$, where r_0 is the average growth rate, $\gamma(t)$ is the noise function and ϵ is the intensity of the fluctuation; alternatively, one can add the randomly fluctuating driving force directly to the deterministic growth equations of prey and predator populations without altering any particular parameter, [18, 30]. In the present study we introduce stochastic perturbation terms into the growth equations to incorporate the effect of a randomly fluctuating environment. We assume that stochastic perturbations of the state variables around their steady-state values E^* are of Gaussian white noise type which are proportional to the distances of f, s, v from their steady-state values f^*, s^*, v^* respectively [4]. Gaussian white noise is extremely useful to model rapidly fluctuating phenomena. We thus obtain the following stochastic system

$$\begin{aligned} df &= F(f, s, v)dt + \sigma_1(f - f^*)d\xi_t^1 \equiv \tilde{F} \\ ds &= S(f, s, v)dt + \sigma_2(s - s^*)d\xi_t^2 \equiv \tilde{S} \\ dv &= P(f, s, v)dt + \sigma_3(v - v^*)d\xi_t^3 \equiv \tilde{P} \end{aligned} \tag{3}$$

where σ_i , $i = 1, 2, 3$ are real constants and known as the intensities of environmental fluctuations, $\xi_t^i = \xi_i(t)$, $i = 1, 2, 3$ are standard Wiener processes, independent of each other [11].

We consider (3) as an Ito stochastic differential system of type

$$dX_t = \chi(t, X_t)dt + g(t, X_t)d\xi_t, \quad X_{t_0} = X_0, \quad (4)$$

where the solution $\{X_t, t > 0\}$ is a Ito process, χ is the slowly varying continuous component or drift coefficient and g is the rapidly varying continuous random component or diffusion coefficient. Here ξ_t denotes a three-dimensional stochastic process having scalar Wiener process components with increments $\Delta\xi_t^j = \xi_{t+\Delta t}^j - \xi_t^j$ which are independent Gaussian random variables $\mathbf{N}(0, \Delta t)$. In formulae,

$$X_t = (f, s, v)^T, \quad \xi_t = (\xi_t^1, \xi_t^2, \xi_t^3)^T, \quad \chi = (F, S, P)^T, \\ g = \begin{pmatrix} \sigma_1(f - f^*) & 0 & 0 \\ 0 & \sigma_2(s - s^*) & 0 \\ 0 & 0 & \sigma_3(v - v^*) \end{pmatrix}. \quad (5)$$

Since the diffusion matrix g depends upon the solution of X_t , the system (3) is said to have multiplicative noise. The stochastic differential system (3) can be centered at its positive equilibrium point $E^*(f^*, s^*, v^*)$ by introducing $u(t) = (u_1(t), u_2(t), u_3(t))^T$ where the new variables are $u_1 = f - f^*$, $u_2 = s - s^*$, $u_3 = v - v^*$. To simplify the mathematics we deal with the stochastic differential equations obtained by linearizing (5) around the coexistence equilibrium E^* . The linearized version of (4) at E^* is given by

$$du(t) = H(u(t))dt + g(u(t))d\xi(t), \quad (6)$$

where

$$H(u(t)) = \left(-\frac{rf^*}{W}u_1 - cf^*u_2, kcs^*u_1 - kbs^*u_3, -bv^*u_2 - \frac{ev^*}{V}u_3 \right)^T, \\ g(u(t)) = \begin{pmatrix} \sigma_1u_1 & 0 & 0 \\ 0 & \sigma_2u_2 & 0 \\ 0 & 0 & \sigma_3u_3 \end{pmatrix}.$$

Note that in (6) the equilibrium E^* corresponds to the origin in the new coordinates $(u_1, u_2, u_3) = (0, 0, 0)$. Let us define the set $\Gamma = \{(t \geq t_0) \times R^3, t_0 \in R^+\}$. Let $Z \in C^3(\Gamma)$ be a continuously differentiable function of time t and twice continuously differentiable function of u . Let us introduce the following operator acting on Z defined by

$$LZ(t, u) = \frac{\partial Z(t, u(t))}{\partial t} + H^T(u(t))\frac{\partial Z(t, u)}{\partial u} + \frac{1}{2}\text{tr}[g^T(u(t))\frac{\partial^2 Z(t, u)}{\partial u^2}g(u(t))], \quad (7)$$

where $\frac{\partial Z}{\partial u} = \left(\frac{\partial Z}{\partial u_1}, \frac{\partial Z}{\partial u_2}, \frac{\partial Z}{\partial u_3} \right)^T$, $\frac{\partial^2 Z(t, u)}{\partial u^2} = \left(\frac{\partial^2 Z}{\partial u_j \partial u_i} \right)$ for $i, j = 1, 2, 3$ and $\text{tr}[A]$ denotes the trace of the matrix A .

To analyze the stability, we need a result [1] which we write in a particular case as follows

Proposition. Suppose for $i = 1, 2, 3$ for suitable $K_i > 0$ the function $Z(t, u) \in C^3(\Gamma)$ satisfies the inequalities

$$K_1|u|^2 \leq Z(t, u) \leq K_2|u|^2, \quad LZ(t, u) \leq -K_3|u|^2. \quad (8)$$

Then the trivial solution of (6) is exponentially mean square stable for all time $t \geq 0$. Furthermore, the trivial solution of (6) is globally asymptotically stable in probability.

We can now prove the following result.

Theorem. Assume that for some positive real values of ω_j , for $j = 1, 2$ the following inequality holds:

$$(2bv^*\omega_2 - \Omega_1\sigma_2^2)(2kbs^*\omega_2 + (\frac{2ev^*}{V} - \sigma_3^2)\Omega_2) > (cf^*\omega_1 + kbs^*\Omega_1 + bv^*\Omega_2)^2, \tag{9}$$

$$\Omega_1 = 1 + \omega_1, \quad \Omega_2 = \omega_1 + \omega_2.$$

Then the zero solution of system (4) is asymptotically mean square stable if

$$\sigma_1^2 < \frac{2rf^*}{W}, \quad \sigma_2^2 < \frac{2bv^*\omega_2}{1 + \omega_2}, \quad \sigma_3^2 < \frac{2kbs^*\omega_2}{\omega_1 + \omega_2} + \frac{2ev^*}{V} \tag{10}$$

where

$$\omega_1^* = \frac{kcs^*}{\Delta}, \quad \omega_2^* = \frac{A}{\Delta}, \quad \Delta = f^*c - A + v^*b > 0, \quad A = \frac{rf^*}{W} + \frac{ev^*}{V}. \tag{11}$$

Proof: Let us consider the Lyapunov function

$$Z(t, u) = \frac{1}{2}[\omega_1(u_1 + u_3)^2 + u_2^2 + \omega_2(u_2 + u_3)^2] \tag{12}$$

where $\omega_i, i = 1, 2$ are real positive constants to be chosen later. It is easy to check that inequalities (8) are true. Furthermore

$$\begin{aligned} LZ(t, u) &= -\frac{rf^*\omega_1}{W}u_1^2 - bv^*\omega_2u_2^2 - (kbs^*\omega_2 + \frac{ev^*}{V}(\omega_1 + \omega_2))u_3^2 \\ &- \left(cf^*\omega_1 + kbs^*(1 + \omega_2) + bv^*(\omega_1 + \omega_2) + \frac{ev^*}{V}\omega_2 \right) u_2u_3 \\ &+ \left(kcs^*\omega_2 - \left(\frac{rf^*}{W} + \frac{ev^*}{V} \right) \omega_1 \right) u_1u_3 \\ &+ (kcs^*(1 + \omega_2) - (cf^* + bv^*)\omega_1) u_1u_2 \\ &+ \frac{1}{2}tr[g^T(u) \frac{\partial^2 Z}{\partial u^2} g(u)] \end{aligned} \tag{13}$$

Now let us remark that

$$\frac{\partial^2 Z}{\partial u^2} = \begin{bmatrix} \omega_1 & 0 & \omega_1 \\ 0 & 1 + \omega_2 & \omega_2 \\ \omega_1 & \omega_2 & \omega_1 + \omega_2 \end{bmatrix}$$

and hence

$$g^T(u) \frac{\partial^2 Z}{\partial u^2} g(u) = \begin{bmatrix} \omega_1\sigma_1^2u_1^2 & 0 & \omega_1\sigma_1\sigma_3u_1u_3 \\ 0 & (1 + \omega_2)\sigma_2^2u_2^2 & \omega_2\sigma_2\sigma_3u_2u_3 \\ \omega_1\sigma_1\sigma_3u_1u_3 & \omega_2\sigma_2\sigma_3u_2u_3 & (\omega_1 + \omega_2)\sigma_3^2u_3^2 \end{bmatrix}$$

with

$$\frac{1}{2}Tr[g^T(u) \frac{\partial^2 Z}{\partial u^2} g(u)] = \frac{1}{2}[\omega_1\sigma_1^2u_1^2 + (1 + \omega_2)\sigma_2^2u_2^2 + (\omega_1 + \omega_2)\sigma_3^2u_3^2]. \tag{14}$$

If in (13) we make the choice (11) from (14) we obtain

$$\begin{aligned} LZ(t, u) &= -\left(\frac{rf^*}{W} - \frac{1}{2}\sigma_1^2\right)\omega_1u_1^2 - (bv^*\omega_2 - \frac{1}{2}(1 + \omega_2)\sigma_2^2)u_2^2 \\ &- (kbs^*\omega_2 + \frac{ev^*}{V}(\omega_1 + \omega_2) - \frac{1}{2}(\omega_1 + \omega_2)\sigma_3^2)u_3^2 \\ &- \{cf^*\omega_1 + kbs^*(1 + \omega_2) + bv^*(\omega_1 + \omega_2) + \frac{ev^*}{V}\omega_2\}u_2u_3 \\ &= -u^TQu \end{aligned}$$

where

$$Q = \begin{bmatrix} \frac{rf^*}{W} - \frac{1}{2}\sigma_1^2 & 0 & 0 \\ 0 & Q_{22} & Q_{23} \\ 0 & Q_{32} & Q_{33} \end{bmatrix},$$

where

$$\begin{aligned} Q_{22} &= bv^*\omega_2 - \frac{1}{2}(1 + \omega_2)\sigma_2^2, \\ Q_{23} &= \frac{1}{2}[cf^*\omega_1 + kbs^*(1 + \omega_2) + bv^*(\omega_1 + \omega_2) + \frac{ev^*}{V}\omega_2] = Q_{32}, \\ Q_{33} &= kbs^*\omega_2 + \frac{v^*}{V}(\omega_1 + \omega_2) - \frac{1}{2}(\omega_1 + \omega_2)\sigma_3^2. \end{aligned}$$

From (9) and (10) we find that Q is a real symmetric positive definite matrix so that all its eigenvalues $\lambda_i(Q)$, $i = 1, 2, 3$, are positive real numbers. Let $\lambda_m = \min\{\lambda_i(Q), i = 1, 2, 3\} > 0$. It follows then

$$LZ(t, u) \leq -u^T q u \leq -\lambda_m |u(t)|^2, \quad (15)$$

thus completing the proof.

We have already observed that the deterministic interior equilibrium point E^* is always stable whenever it exists. We observe that E^* remains stable under the stochastic perturbation if and only if the condition stated in the Theorem hold. In other words, we may conclude that the deterministically stable interior equilibrium point E^* may become unstable if the intensity of the environment exceeds the limiting values given in (10). This fact will be investigated in detail in the next Section in the numerical simulations.

4 Discussion

4.1 The models without human intervention

We have carried out numerical simulations to check some of the theoretical results. For the parameter values $a = 3.1$, $b = 1.2$, $c = .2$, $e = 2.5$, $r = 1$, $W = 5$, $V = 400$, $k = 1$ we observe that the system (1) is locally asymptotically stable around the coexistence steady state $E^* \equiv (2.92, 2.08, 2.1)$, see Figure 1. From our analytical result we observe that if E^* is locally asymptotically stable in the absence of environmental fluctuations, it remains so even in the presence of environmental disturbances provided the intensity of the environmental fluctuations is under a certain threshold value. The latter relative to the above set of parameter values as calculated in the Theorem are $\sigma_1 = 1.08$, $\sigma_2 = 1.94$ and $\sigma_3 = 2.15$. The result which we obtain is confirmed from Figure 2 and Figure 4. Figure 2, with $\sigma_1 = 0.1$, $\sigma_2 = 0.5$ and $\sigma_3 = 0.6$ which are below the calculated threshold value, shows that E^* is stochastically stable because if we imagine a circular or elliptic neighborhood around the equilibrium point E^* , we find that 90% or more of the population distribution lies within the said neighborhood, see Figure 3. From Figure 4, with parameter values violating the maximum threshold values (10) given in the Theorem, $\sigma_1 = 1.3$, $\sigma_2 = 2.1$, $\sigma_3 = 2.5$, we observe that there is an increase in the fluctuations.

4.2 Effect of Spraying

Human intervention in the ecosystem is considered allowing insecticide spraying. We have to distinguish several situations.

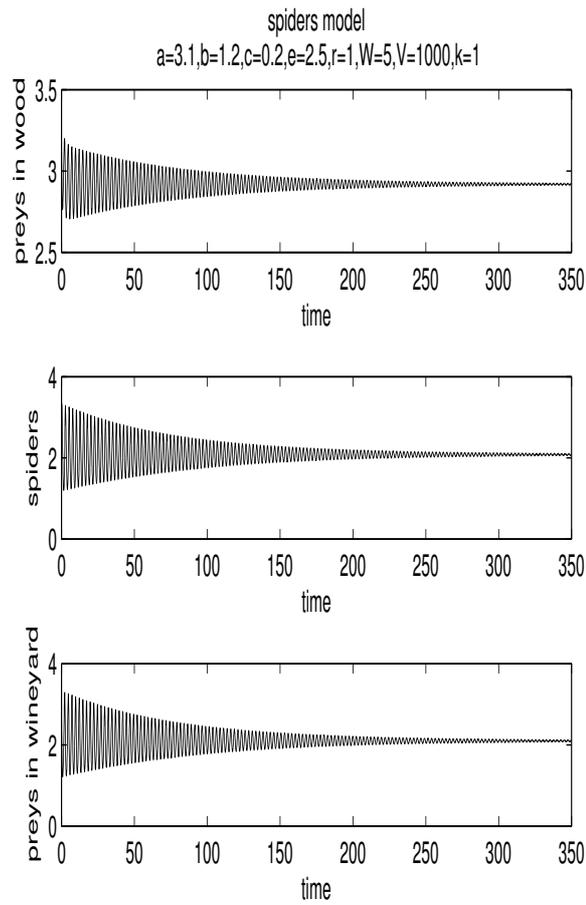


Figure 1: Interior equilibrium point E^* is stable for $a = 3.1$, $b = 1.2$, $c = .2$, $e = 2.5$, $r = 1$, $W = 5$, $V = 400$, $k = 1$.

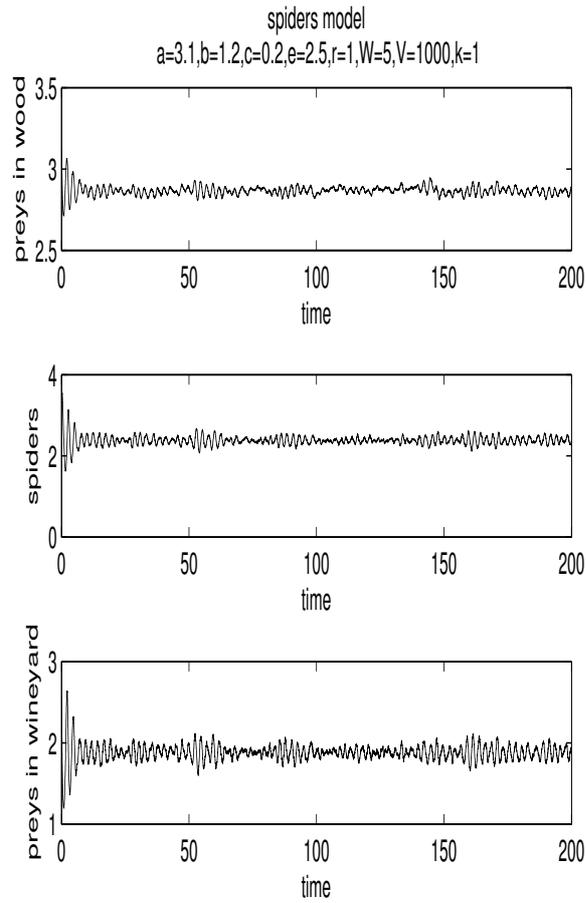


Figure 2: Interior equilibrium point E^* is stable under the stochastic perturbation for $\sigma_1 = 0.1$, $\sigma_2 = 0.5$, $\sigma_3 = 0.6$.

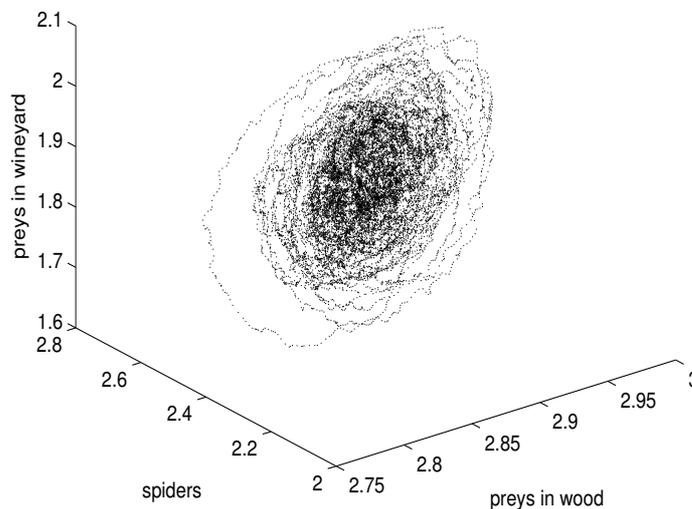


Figure 3: Phase space diagram depicting the stability of E^* under stochastic perturbation satisfying the Theorem.

In high input agriculture, chemical treatments with insecticides are used against *flavescence dorée*, also anti-rot compounds are employed, the major part of treatments being with dicarboximidic fungicides like Procymidone against *Botrytis cinerea*. Other products are carbaryl, methyl parathion, dicofol, azinphos-methyl, phosmet, fenbutatin-oxide, [23], phosalone, [22], azinphos-methyl, fenpropathrin, imidacloprid, azadirachtin, capsaicin, caolin [15]. Other techniques involve the use of sulphur spraying, copper, zinc, products with esaconazol and copper oxiclourur sulphate against oidium and rots, carbamate pesticides and fungicide (mancozeb, benalaxil, triadimeno, cymoxanil), [6] mean use of mineral feeds with P, Ka e N of 4.5 q/ha. Notice that in high input agriculture the herbaceous layer is not always present during the whole year, while grass mowing and tilling are performed at most twice a year, as needed.

For biological agriculture the European Council Regulation 2092/91 allows chemical treatments with pre- and post- emergence herbicides, mostly glufosinate; insecticides, prevalently against *flavescence dorée*; anti-rot compounds, sulphur spraying, copper, zinc, products with esaconazol and copper oxiclourur sulphate against oidium and rots, carbamate pesticides and fungicides other than Mancozeb, mean use of mineral feeds with P, Ka e N of 6.5 q/ha. It should be remarked that these agricultural practices reduce dramatically the grass cover because of the use of herbicides and tilling.

Certified organic production instead does not allow chemical treatments other than sulphur and copper sulfate spraying. In some cases pyrethrum is sprayed against *flavescence dorée*. The presence of the herbaceous layer is constant over the whole year. Depending on the need, grass is mowed once or twice a year.

The action of spraying can be modelled by considering an impulse function, at particular instants in time, thus obtaining

$$\dot{f} = F - h(1 - q)\delta(t_i), \quad \dot{s} = S - hKq\delta(t_i), \quad \dot{v} = V - hq\delta(t_i), \quad (16)$$

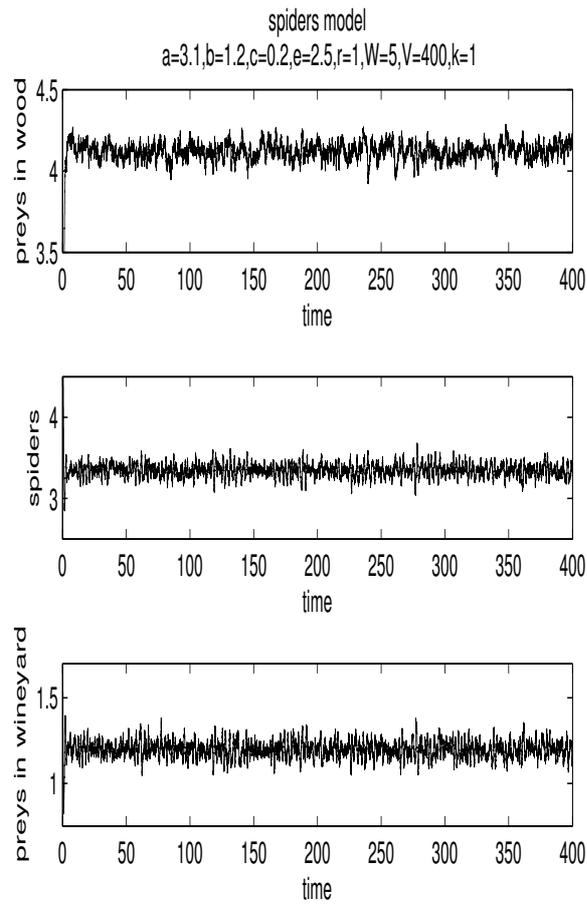


Figure 4: Effects of increased stochastic fluctuations past the thresholds values of the Theorem, $\sigma_1 = 1.3$, $\sigma_2 = 2.1$, $\sigma_3 = 2.5$.

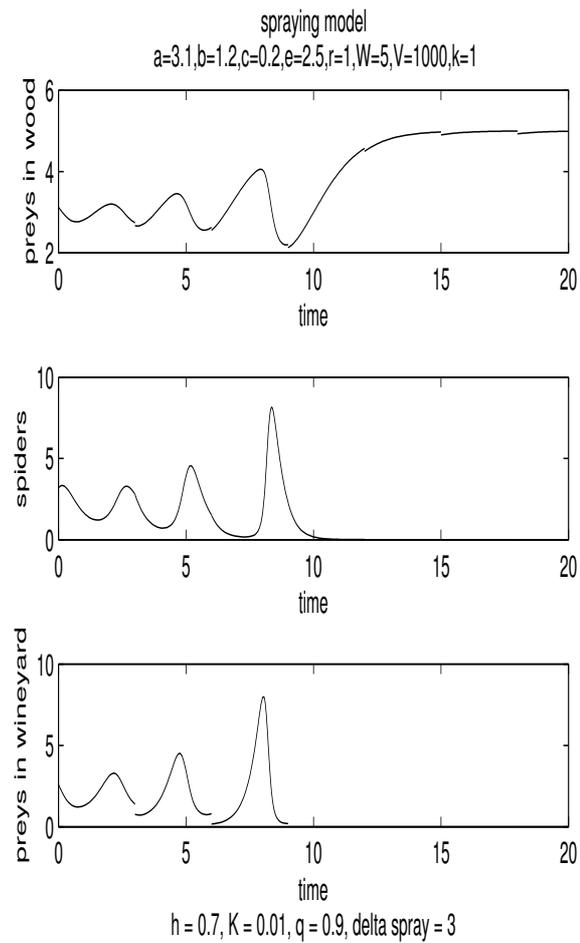


Figure 5: Spraying effects in the absence of the environmental fluctuation: all vineyard insects go extinct.

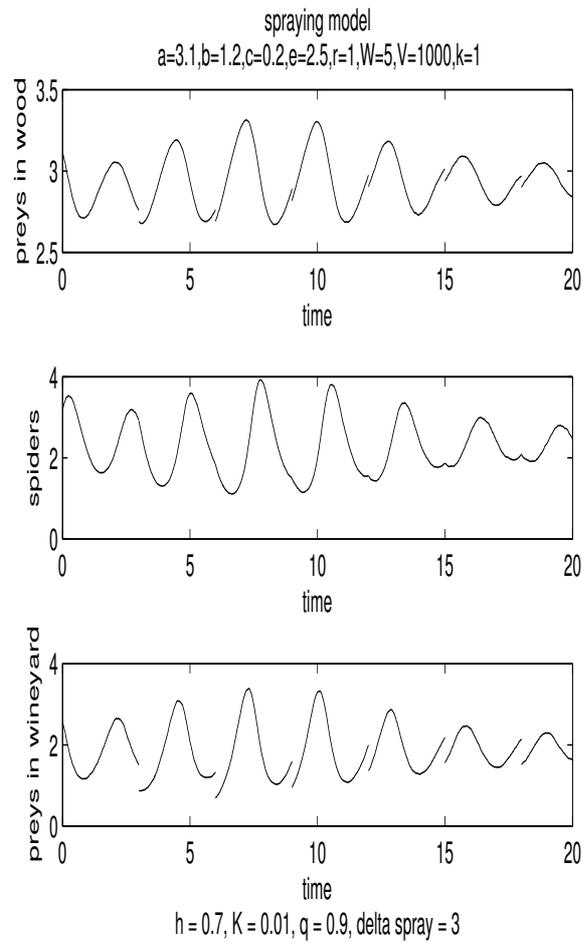


Figure 6: Spraying effects in the presence of the environmental fluctuation: vineyard insects survive in the system.

where $\delta(t_i)$ denotes the Dirac's delta function and t_i , $i = 1, 2, \dots$ are the spraying instants. Moreover q denotes the portion of insecticide sprayed directly onto the vineyards, $1 - q$ the portion instead being dispersed in the woods, h and $0 < K < 1$ represent respectively the effectiveness against the parasites and the reduced action on the spiders. The numerical experiment reported in Figure 5 has been performed on the deterministic system (1) with the same demographic parameters given in Figure 1, with constants $h = 0.7$, $K = 0.01$, $q = 0.9$ and an interval 3 between spraying times. The impact of spraying is immediately evident, as the vineyard pests are wiped out, but so are also the spiders, although the spraying effectiveness level on the latter is extremely low. This result may apparently be satisfactory from the farmer's viewpoint, but it is a disaster for biological diversity, as the ecosystem is impoverished. Wood insects apparently survive.

To see the spraying effect on the ecosystem in the presence of environmental disturbances, we need to incorporate the spraying effect in our system (3) to obtain the new system as

$$\dot{f} = \tilde{F} - h(1 - q)\delta(t_i), \quad \dot{s} = \tilde{S} - hKq\delta(t_i), \quad \dot{v} = \tilde{P} - hq\delta(t_i), \quad (17)$$

We observe in Figures 5 and 6 that with the parameters σ_i as given in Figure 2, and once again the demographic parameters being the same as those of Figure 1, the vineyard pests which were wiped out in the deterministic environment, Figure 5, persist instead in the environment with variable conditions, Figure 6.

5 Conclusions

The models presented here induce the following considerations. The vineyard pests survive in the ecosystem even after spraying, thus in the fluctuating environment we may need to increase the amount of spraying to control them, but this action may have adverse effects from the ecological viewpoint. Indeed spraying does not seem to be a good option, since field studies support the statement that pesticide treatment increases some pest populations with outbreaks that may be relevant, [14, 25]. The latter seem to be the consequence of predators, i.e. spiders, reduction, due to the negative side effects of chemical spraying. Spiders seem in fact to be more sensitive to pesticides than phytophagous pests [19]. These adverse effects are even stronger in a variable environment. Our study supports the need of some kind of biological control on top or in addition to spraying, a role which may well be played by the spiders. Finally in order to maintain the ecosystem in stable conditions, the environment fluctuations must remain within certain threshold values, otherwise the system even if deterministically stable, under variable perturbations may become unstable with possibly unpredictable consequences. This may suggest that weather conditions and climate changes could also have an important effect on these ecosystems.

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