# Hexapod Robots: New CPG Model for Generation of Trajectories [1]

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*Abstract:* Legged robots are often used in a large variety of tasks, in different environments. The large number of degrees-of-freedom, to be controlled during these tasks, turns the online generation of trajectories in these robots very complex. In this paper, we consider a modular approach to online generation of trajectories, based on biological concepts, namely Central Pattern Generators (CPGs). We introduce a new CPG model for hexapod robots' rhythms, that generalizes the work of Golubitsky, Stewart, Buono and Collins (1998,1999). Each neuron/oscillator in the CPG consists of two modules/primitives: rhythmic and discrete, that are modeled by nonlinear dynamical systems. Superposition of discrete and rhythmic primitives permits the modeling of complex motor behaviors, namely locomotion in irregular terrain and obstacle avoidance. We study the effect on the amplitude and frequency of the robots' gaits of superimposing the two motor primitives. The discrete primitive is inserted as an offset of the solution of the rhythmic primitive. We also consider three types of couplings between CPG units: synaptic, diffusive and mixed. Simulation results reveal interesting facts, in certain cases amplitude and frequency of periodic solutions, identified with hexapods' tripod, caterpillar and metachronal gaits, remain constant. Therefore, it is possible to use these solutions to generate trajectories for the joint values of real six-legged robots, since varying the joint offset will not affect the required amplitude and frequency of the resultant trajectory nor the gait.

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# 1 Introduction

In the last few years, there has been a large development in the modeling of online generation of trajectories in legged robots. Models producing rhythmic robots' patterns, inspired in biology, are now widely spread. Locomotion in vertebrates is commonly structured in three layers [19, 11]. The top layer consists of the brainstem command systems, the structures that decide which motor pattern is to be activated at each moment of time. The middle layer is composed of the steering and posture control systems. The bottom layer includes the Central Pattern Generators (CPGs). CPGs are networks of neurons, presumably located at the spinal level of vertebrates, that are responsible for the rhythmic patterns observed during animals' locomotion. Mathematically, CPGs are commonly modeled by coupled nonlinear dynamical systems [8, 9, 5].

Dynamical systems have very nice properties, such as smooth modulation, low computational cost, phase-locking between oscillators, that are extremely useful to online modulation of trajectories [17, 4].

Schöner *et al* [27] propose a set of organizational principles that allow an autonomous vehicle to perform stable planning. In 1994, Lewis *et al* [12], propose a complex motor pattern (CMP) generator for the control of the leg movements of a hexapod robot. The CMP consists of a network of neurons and the interconnection weights between the neurons are optimized by a Genetic Algorithm (GA). In the GA, the designer specifies "islands of fitness" on the way to the final goal. The system itself determined how to evolve from one island to the next through the GA. Yang *et al* [30] studied a biologically inspired neural network model to real-time collision-free motion planning of mobile robots or manipulators in a non stationary environment. In [25], authors design, model and control an hexapod robot, RHex. RHex has six actuated degrees of freedom, located at the hip. Simulations show that RHex is capable of stable walking, running and turning. The control of the robot is achieved by the use of finite state approaches. In 2001, Schöner and Santos [28] propose an attractor based two-layer dynamics for autonomous vehicles when relatively low-level, noisy sensorial information is used to steer action. This architecture autonomously generates timed movements and sequences of movements stably adapted to online changes of sensory information. In 2002, Arena *et al* [1] propose a CPG for hexapod robots that includes sensorial feedback. The CPG is modeled by cellular neural networks (CNNs) and local bifurcation of the CNN cells allow control of the robot. That is to say that changing the value of the bias of CNN cells produces suitable control of the robot. CNN for hexapod locomotor control are also studied in [14], where a new design of a CNN is presented. Authors introduce new state equations, reducing the number of CNN cells, from 12 to 6, and lowering the complexity of the CNN circuit. Santos [24] develops previous work ([28]) and integrates a timing architecture with another dynamical architecture, which does not explicitly parameterize timing requirements. The timing model consists of a two layer architecture, in which a competitive neural dynamics controls the qualitative dynamics of a second, timing layer. At that second layer, periodic attractors generate timed movement. By switching between the limit cycle and the fixed points, discrete movements and sequences of movements are obtained. These switches are controlled by the parameters of the neural dynamics which express sensory information and logical conditions. Dégallier *et al* [7] use a dynamical systems' approach to do online generation of trajectories in a robot performing a drumming task. Makarov*et al* [16] propose a Toda-Rayleigh (TR) ring, consisting of six TR units, as a model for a CPG for controlling hexapodal robots. Common hexapodal gaits, such as tripod and caterpillar, are reproduced by this CPG model when coupled to the limb actuators. Authors also implement electronically the CPG and show how to incorporate the actuator (motor) dynamics in it. The approach used in this paper allows adaptability of locomotion in nonhomogeneous environments, without the need of higher level intervention. Matos *et al* [17] propose a bio-inspired robotic controller able to generate locomotion and to easily switch between different types of gaits. Oliveira *et al* [18] develop a quadruped locomotion controller able to generate locomotion and reaching visually acquired markers. Campos *et al* [4], present a two-layer architecture to model hexapod robots' locomotion. The bottom layer consists of the CPG for generating hexapods' gaits and the second layer sets up the parameter values for each gait. They study smooth gait transition in the model, using a modulatory drive signal regulating CPG's activity. Authors also propose a lateral posture control, based on

dynamical systems, that corrects the robot posture and keeps its balance, when subject to changes in the lateral tilt.

Pinto *et al* [22] introduce a new CPG model for the generation of hexapods robots' gaits that consists of twelve oscillators, two for each leg. The trajectories are composed of two distinct primitives, rhythmic and discrete. Authors study the effect of superimposing these primitives on the robot's gaits, when the discrete primitive is inserted as an offset of the rhythmic one and two types of coupling are considered: diffusive and synaptic. Numerical results show that amplitude and frequency values of the periodic solutions produced by the CPG model and identified with tripod and metachronal are almost constant for the two couplings.

In this paper, we develop the work done by Pinto *et al* [22]. We consider the CPG model hexapod-robot, in Fig 1, for modular generation of an hexapod robot movements, using a biological approach [2]. CPG hexapod-robot is a network of twelve coupled CPG-units, each of which consists of two motor primitives: rhythmic and discrete. The periodic solutions produced by the CPG model hexapod-robot are identified with common hexapodal gaits, such as tripod, metachronal caterpillar pronk, and lurch. We study the variation in the amplitude and the frequency values of these periodic solutions when the two primitives are superimposed. The discrete primitive is inserted as an offset of the purely rhythmic periodic solution. We consider three types of coupling between the CPG units, diffusive, synaptic, and mixed. The main goal is to show that these discrete corrections may be performed since that they do not affect the gait, meaning that the amplitude and frequency of the resultant trajectories is kept constant. Therefore, it is possible to use the solutions to generate trajectories for the joint values of real robots, since varying the joint offset will not affect the required amplitude and frequency of the resultant trajectory, nor the gait. Amplitude and frequency may be identified, respectively, with the range of motion and the velocity of the robot's movements, when considering implementations of the proposed controllers for generating trajectories for the joints of rea l robots.

## 2  CPG model for hexapods

In this section we introduce the CPG model for online generation of trajectories of hexapod robots, hexapod-robot. It is based in the work of Golubitsky *et al* [9, 10]. We give the general class of systems of ordinary differential equations (ODEs) that model CPG hexapod-robot and resume the symmetry techniques that allow classification of periodic solutions produced by this CPG model and identified with common hexapod locomotor rhythms.

### 2.1  Architecture and symmetry

Figure 1 shows the CPG model hexapod-robot(Fig 1) for generating locomotion for hexapods robots. It consists of twelve coupled oscillators. The oscillators (or cells) are denoted by circles and the arrows represent the couplings between cells. All cells are identical, since they are represented by the same symbol. There are two distinct arrows that represent two distinct coupling strengths.

Each cell is a CPG unit and is divided into two motor primitives, discrete and rhythmic, both modeled by nonlinear dynamical systems.

This is the minimal CPG capable of producing all of the hexapodal gaits (Table 2). The choice of a 12-cells CPG model opposed to a 6-cells model may be seen as a increase of the complexity of the model, associated with more integration time and lower speed. Nevertheless, the repeated advances in computer speed and the availability of fast, optimized software 'engines', to implement simulations of real world phenomena, turn hexapod-robot into a viable solution for trajectory generation in robotics.
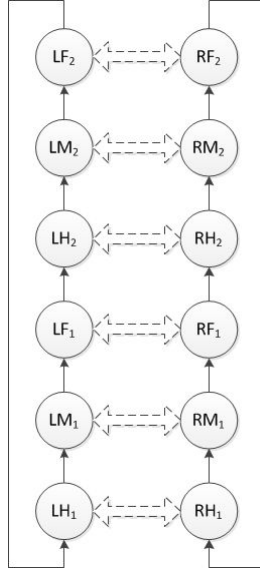
Figure 1: CPG locomotor model for hexapods, hexapod-robot. LF (left fore leg), LM (left middle leg), LH (left hind leg), RF (right fore leg), RM (right middle leg), RH (right hind leg).

## 2.2 General class of ODEs

The general class of systems of ODEs modeling CPG model hexapod-robot is:

$$
\begin{aligned}
\dot{x}_{LH_1} &= F(x_{LH_1}, x_{RH_1}, x_{LF_2}) \\
\dot{x}_{RH_1} &= F(x_{RH_1}, x_{LH_1}, x_{RF_2}) \\
\dot{x}_{LM_1} &= F(x_{LM_1}, x_{RM_1}, x_{LH_1}) \\
\dot{x}_{RM_1} &= F(x_{RM_1}, x_{LM_1}, x_{RH_1}) \\
\dot{x}_{LF_1} &= F(x_{LF_1}, x_{RF_1}, x_{LM_1}) \\
\dot{x}_{RF_1} &= F(x_{RF_1}, x_{LF_1}, x_{RM_1}) \\
\dot{x}_{LH_2} &= F(x_{LH_2}, x_{RH_2}, x_{LF_1}) \\
\dot{x}_{RH_2} &= F(x_{RH_2}, x_{LH_2}, x_{RF_1}) \\
\dot{x}_{LM_2} &= F(x_{LM_2}, x_{RM_2}, x_{LH_2}) \\
\dot{x}_{RM_2} &= F(x_{RM_2}, x_{LM_2}, x_{RH_2}) \\
\dot{x}_{LF_2} &= F(x_{LF_2}, x_{RF_2}, x_{LM_2}) \\
\dot{x}_{RF_2} &= F(x_{RF_2}, x_{LF_2}, x_{RM_2})
\end{aligned}
\tag{1}
$$

where $x_i \in \mathbf{R}^k$ is the cell $i$ variables, $k$ is the dimension of cells' internal dynamics, and $F : (\mathbf{R}^k)^3 \to \mathbf{R}^k$, is an arbitrary mapping. We assume that and all cells/neurons are identical, i.e, all cells have the same internal dynamics.

CPG hexapod-robot has

$$
\Gamma_{\mathsf{hexapod-robot}} = \mathbf{Z}_6(\omega) \times \mathbf{Z}_2(\kappa)
$$

symmetry group. The CPG model hexapod-robot has the bilateral symmetry of animals ($\mathbf{Z}_2(\kappa)$) and a translational symmetry ($\mathbf{Z}_6(\omega)$), from back to front, i.e, RF is coupled to cell RH, and the same applies for cells on the left side. $\mathbf{Z}_2(\kappa)$ is the cyclic group of order 2, and $\mathbf{Z}_6(\omega)$ is the cyclic group of order 6.

### *2.2.1 Is symmetry reasonable?*

Symmetry has been used to build CPG models for locomotion of animals and robots [10, 8, 23, 13, 22, 20, 21]. This need to include symmetry is related with the phase relations between legs' movements in animals. Some examples follow. In the biped walk, where left and right legs are half-period out of phase, or in the hexapod metachronal gait, where left and right legs are half-period out of phase. The major phase shifts observed in locomotor rhythms are simple fractions of the period, suggesting that the model CPG might have some symmetry, since these particular phase shifts are unusual in asymmetric systems. Additionally, bilateral symmetry is prevalent in the animal kingdom, thus it seems reasonable that a CPG model for animal locomotor patterns should have at least bilateral symmetry.

Symmetry seems, thus, a reasonable modeling assumption, even though real life animals are not perfectly symmetric.

## 2.3 Periodic solutions and hexapodal gaits

Theorem $H/K$ [8] allows the identification of symmetry types of periodic solutions, produced by a given coupled cell network. These periodic solutions are then identified with animals locomotor rhythms. Let $x(t)$ be a periodic solution of an ODE $\dot{x} = f(x)$, with period normalized to 1, and with symmetry group $\Gamma$. Let $H$ and $K$ be subgroups of $\Gamma$. Spatial symmetries $K$ fix the solution pointwise, i.e., let $\gamma \in \Gamma$, then $\gamma x(t) = x(t)$. Spatio-temporal symmetries $H$ fix the solution setwise, i.e., $\gamma x(t) = x(t - \theta) \leftrightarrow x(t + \theta) = x(t)$, where $\theta$ is the phase shift associated to $\gamma$. If $\theta = 0$, then $\gamma$ is a spatial symmetry.

In order for $(H, K)$ to correspond to symmetries of a periodic solution $x(t)$ to (1), for some function $F$, the quotient $H/K$ must be cyclic. In Table 1, we show those pairs of symmetry types $(H, K)$ such as $H/K$ is cyclic and their identification with common hexapod rhythms, such as pronk, lurch, metachronal, and tripod. Table 2, exhibits the corresponding periodic solutions.

Some gaits in Tables 1-2, such as the rolling tripod or the inchworm, have not yet been observed in real hexapods, nevertheless, it is worth verifying whether they are applicable to six-legged robots. Robots' dynamics may be inspired in biology but do not have to mimic all of real animals' behavior.

| $H$ | $K$ | **Gait** |
|---|---|---|
| $\Gamma_{\text{hexapod−robot}}$ | $\Gamma_{\text{hexapod−robot}}$ | pronk |
| $\Gamma_{\text{hexapod−robot}}$ | $\mathbf{Z}_6(\omega\kappa)$ | tripod |
| $\Gamma_{\text{hexapod−robot}}$ | $\mathbf{Z}_6(\omega)$ | pace |
| $\Gamma_{\text{hexapod−robot}}$ | $\mathbf{D}_3(k,\omega^3)$ | caterpillar |
| $\Gamma_{\text{hexapod−robot}}$ | $\mathbf{Z}_3(\omega^2\kappa)$ | lurch |
| $\Gamma_{\text{hexapod−robot}}$ | $\mathbf{Z}_2(\omega^3\kappa)$ | metach. |
| $\Gamma_{\text{hexapod−robot}}$ | $\mathbf{Z}_2(\kappa)$ | inchworm |
| $\Gamma_{\text{hexapod−robot}}$ | $\mathbf{Z}_2(\omega^3)$ | rolling tripod |

Table 1: Symmetry pairs of periodic solutions, produced by the coupled cells system (1), and corresponding gaits.

The tripod is a gait used for fast displacements. In the tripod, the left hind and fore legs, and the contralateral middle leg move together. This is also valid for the remaining three legs. Between these two groups of legs, there is a phase shift of $1/2$ period. The tripod is identified with a periodic solution of CPG hexapod-robot with symmetry pair $(H,K) = (\Gamma_{\text{hexapod−robot}}, \mathbf{Z}_6(\omega\kappa))$. Consider $x(t)=(x_1(t),...,x_{12}(t))$ a periodic solution produced by CPG model hexapod-robot. In order to understand transformation $\omega\kappa$, we first apply $\kappa$ to $x(t)$, obtaining:

$$\hat{x}(t) = (x_2(t),x_1(t),x_4(t),x_3(t),x_6(t),x_5(t),x_8(t),x_7(t),x_{10}(t),x_9(t),x_{12}(t),x_{11}(t))$$

| Left | Middle | Right | Gait |
|:---:|:---:|:---:|:---:|
| $(x_{LH},x_{LH})$ | $(x_{LH},x_{LH})$ | $(x_{LH},x_{LH})$ | pronk |
| $(x_{LH},x_{LH}^S)$ | $(x_{LH}^S,x_{LH})$ | $(x_{LH},x_{LH}^S)$ | tripod |
| $(x_{LH},x_{LH}^S)$ | $(x_{LH},x_{LH}^S)$ | $(x_{LH},x_{LH}^S)$ | pace |
| $(x_{LH},x_{LH})$ | $(x_{LH}^{2S/3},x_{LH}^{2S/3})$ | $(x_{LH}^{4S/3},x_{LH}^{4S/3})$ | caterpillar |
| $(x_{LH},x_{LH})$ | $(x_{LH}^S,x_{LH}^S)$ | $(x_{LH},x_{LH})$ | lurch |
| $(x_{LH},x_{LH}^S)$ | $(x_{LH}^{S/3},x_{LH}^{4S/3})$ | $(x_{LH}^{2S/3},x_{LH}^{5S/3})$ | metac. |
| $(x_{LH},x_{LH})$ | $(x_{LH}^{S/3},x_{LH}^{S/3})$ | $(x_{LH}^{2S/3},x_{LH}^{2S/3})$ | inch. |
| $(x_{LH},x_{LH}^S)$ | $(x_{LH}^{2S/3},x_{LH}^{5S/3})$ | $(x_{LH}^{4S/3},x_{LH}^{S/3})$ | rol. trip. |

Table 2: Periodic solutions of system (1), identified with common hexapods gaits, where period of solutions is normalized to 1. We only show the bottom six cells, the others can be easily computed. $^S$ is half period out of phase.

After that, we apply $\omega$ to $\hat{x}(t)$, resulting in solution

$$\tilde{x}(t) = (x_4(t),x_3(t),x_6(t),x_5(t),x_8(t),x_7(t),x_{10}(t),x_9(t),x_{12}(t),x_{11}(t),x_2(t),x_1(t))$$

Thus, spatial symmetry $\omega\kappa$ forces the solution to have the form

$$\bar{x}(t) = (x_1(t),x_2(t),x_2(t),x_1(t),x_1(t),x_2(t),x_2(t),x_1(t),x_1(t),x_2(t),x_2(t),x_1(t))$$

Applying $\omega\kappa$ to the tripod does not change that gait since the groups of cells $(1,4,5,8,9,12)$ and $(2,3,6,7,10,11)$ receive the same set of signals. Spatio-temporal symmetries $\Gamma_{\text{hexapod-robot}}$ force signals sent to the two groups of cells above to be equal and to be half period out of phase. Thus, the final solution identified with tripod has the form:

$$\bar{x}(t) = (x_1(t),x_1(t+1/2),x_1(t+1/2),x_1(t),x_1(t),x_1(t+1/2),x_1(t+1/2),x_1(t),x_1(t),x_1(t+1/2),x_1(t+1/2),x_1(t))$$

The caterpillar is used by arthropods for a medium speed locomotion. The ipsilateral legs move with a phase shift of $1/3$ of the period and left and corresponding right legs move together. This gait is identified with a periodic solution of CPG (1) with symmetry group $\left(\Gamma_{\text{hexapod-robot}}, \mathbf{D}_3(\omega^3,\kappa)\right)$.

The metachronal gait is used by six-legged animals for slow locomotor rhythms. This gait can be described as a 'wave' propagating forward from the rear of the animal, first on the left side, and then on the right side. Legs on the same side move with a phase shift of $1/3$ of the period, and right and left legs are $1/2$ period out of phase. This gait is identified with a periodic solution of CPG (1) with symmetry group $\left(\Gamma_{\text{hexapod-robot}}, \mathbf{Z}_2(\omega^3)\right)$.

### 2.4   Stability of hexapodal gaits

Suppose that $x(t)$ is a hyperbolic periodic solution to CPG hexapod-robot, corresponding to a gait with spatiotemporal symmetries $H$ and spatial symmetries $K$, where $K \subset H \subset \Gamma_{\text{hexapod-robot}}$. Theorem 3.6 (page 67) of Golubitsky and Stewart [8] guarantees the existence of a hyperbolic stable periodic solutions to system (1), with spatial symmetry $K$ and spatiotemporal symmetry $H$. Thus, in the case where the discrete part is inserted in all limbs and with equal value, periodic solutions produced by the CPG model hexapod-robot are stable and are identified with the hexapodal gaits in Table 1.

The identification of periodic solutions in Table 1 may also be done using bifurcation theory. To compute stability of those periodic solutions, it is necessary to compute the eigenvalues of the linearization $L$ of system (1) at an equilibrium $X = (x,x,x,x,x,x,x,x,x,x,x,x)$. If the group action is complicated, bare hands computation or computer algebra systems give little insight into the results.

The $\Gamma_{\text{hexapod}-\text{robot}}$ symmetry group of system (1) implies that $L$ commutes with $\Gamma_{\text{hexapod}-\text{robot}}$, and this translates in nice properties on matrix $L$.

We can decompose $\mathbf{P} = (\mathbf{R}^k)^{12}$ into a direct sum of $\Gamma_{\text{hexapod}-\text{robot}}$ irreducible subspaces. In general, this decomposition is not unique, nevertheless, if we use components that combine together all of the $\Gamma_{\text{hexapod}-\text{robot}}$ irreducible subspaces that lie in a fixed isomorphism class, then we obtain a decomposition that is unique. This decomposition is also invariant under $L$.

Let $V_{jk}$ be the sum of all irreducible subspaces that are isomorphic to a representation $\lambda \in \Gamma_{\text{hexapod}-\text{robot}}$. That is, $V_{jk}$ is the isotypic component of $(\mathbf{R}^k)^{12}$, corresponding to $\lambda$. Subspaces $V_{jk}$ are defined as follows. Let $\sigma = e^{i\pi/3}$, then $V_{jk}$ is spanned, over $\mathbf{C}$, by:

$$v_{jk} = \left(1, (-1)^k; \sigma^j, (-1)^k \sigma^j; \sigma^{2j}, (-1)^k \sigma^{2j}; \ldots; \sigma^{5j}, (-1)^k \sigma^{5j}\right)$$

Since $\Gamma_{\text{hexapod}-\text{robot}}$ is abelian, there are eight distinct one-dimensional representations of $\Gamma_{\text{hexapod}-\text{robot}}$. These representations are distinguished by their kernels and are denoted by such. The decomposition of $\mathbf{P}$ into irreducibles is into the representations $P_{jk}$, where

$$P_{jk} = \begin{cases} \text{Re}(V_{jk}) & \text{if } j = 0, 3 \\ \text{Re}(V_{jk} \oplus V_{5-j,k}) & \text{if } j = 1, 2 \end{cases}$$

See Table 3 for a correspondence between gaits in Table 1 and irreducibles $P_{jk}$.

| $P_{jk}$ | Hexapodal gaits |
|---|---|
| $P_{00}$ | pronk |
| $P_{31}$ | tripod |
| $P_{01}$ | pace |
| $P_{20}$ | caterpillar |
| $P_{30}$ | lurch |
| $P_{11}$ | metachronal |
| $P_{10}$ | inchworm |
| $P_{21}$ | rolling tripod |

Table 3: Periodic solutions of CPG hexapod-robot and corresponding isotypic components.

The isotypic decomposition can be used to block-diagonalize $L$ (see Theorem 2.12 in [8]). The later makes easier the task of computing the eigenvalues of $L$. Let $A$, $B$, $C$ be $k \times k$ matrices, where $A$ is the part of the Jacobian $L$ corresponding to the oscillators' internal dynamics, $B$ corresponds to the bilateral coupling, and $C$ is the ipsilateral coupling. Matrix $L$ is given by:

$$L = \begin{bmatrix} A & B & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & C & 0 \\ B & A & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & C \\ C & 0 & A & B & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & C & B & A & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C & 0 & A & B & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C & B & A & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & C & 0 & A & B & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & C & B & A & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & C & 0 & A & B & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & C & B & A & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & C & 0 & A & B \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & C & B & A \end{bmatrix}$$

It follows from Table 3 that the eigenvalues of $L$ are the eigenvalues of block matrices $L_{\Gamma_{\text{hexapod}-\text{robot}}}$, $L_{\mathbf{Z}_6(\omega\kappa)}$, $L_{\mathbf{Z}_6(\omega)}$, $L_{\mathbf{D}_3(\kappa,\omega^3)}$, $L_{\mathbf{Z}_3(\omega^2\kappa)}$, $L_{\mathbf{Z}_2(\omega^k\kappa)}$, $L_{\mathbf{Z}_2(\kappa)}$, and $L_{\mathbf{Z}_2(\omega^3)}$. See (2) for examples.

$$L_{\Gamma_{\text{hexapod}-\text{robot}}} = A+B+C \quad L_{\mathbf{Z}_6(\omega\kappa)} = A-B-C \quad L_{\mathbf{Z}_6(\omega)} = A+B-C \tag{2}$$

As $\Gamma_{\text{hexapod}-\text{robot}}$ is an abelian group, generically, Hopf bifurcation occurs with simple pairs of purely imaginary eigenvalues. Thus, we may compute Hopf bifurcation points for each matrix $L_\lambda$ and stability of the corresponding periodic orbits. Table 4 shows the correspondence between periodic orbits obtained from $L_\lambda$ and hexapodal gaits in Table 1.

| $L_\lambda$ | Hexapodal gaits |
|---|---|
| $L_{\Gamma_{\text{hexapod}-\text{robot}}}$ | pronk |
| $L_{\mathbf{Z}_6(\omega\kappa)}$ | tripod |
| $L_{\mathbf{Z}_6(\omega)}$ | pace |
| $L_{\mathbf{D}_3(\kappa,\omega^3)}$ | caterpillar |
| $L_{\mathbf{Z}_3(\omega^2\kappa)}$ | lurch |
| $L_{\mathbf{Z}_2(\omega^3\kappa)}$ | metachronal |
| $L_{\mathbf{Z}_2(\kappa)}$ | inchworm |
| $L_{\mathbf{Z}_2(\omega^3)}$ | rolling tripod |

Table 4: Periodic solutions of CPG hexapod-robot obtained from Hopf bifurcation of $L_\lambda$.

In all other cases, where the discrete part is inserted in all limbs with unequal values, or is only inserted in some of the limbs, the periodic solution obtained is not any of the hexapodal gaits in Table 4. This is explained by the fact that the symmetry group of this solution $\tilde{x}(t)$ is no longer $\Gamma_{\text{hexapod}-\text{robot}}$, but is a smaller group. This, the solution $\tilde{x}(t)$ might be 'close' to $x(t)$ but is not the same. Mathematically, this can be explained as follows. Suppose that we consider a small forced symmetry-breaking of the equations (1), so that there are distinct functions in (1) modeling the oscillators' internal dynamics. Hyperbolicity implies that a solution $x(t)$ of CPG-hexapod-robot perturbs to a periodic solution of a network close to hexapod-robot, but with oscillators of different types. For instance, you can consider left cells distinct from right cells, so you have function $F_1$ modeling the dynamics of cells on the left, and function $F_2$ modeling the dynamics of cells on the right, with $F_1 \neq F_2$. Let's call this new network hexapod-robot-2. The symmetry group of hexapod-robot-2 is $\Gamma_{\text{hexapod}-\text{robot}-2} = \mathbf{Z}_6(\omega) \subset \Gamma_{\text{hexapod}-\text{robot}}$.

The perturbed solution $\tilde{x}(t)$ has symmetry groups $H' = H \cap \Gamma_{\text{hexapod}-\text{robot}-2}$ and $K' = K \cap \Gamma_{\text{hexapod}-\text{robot}-2}$. These points are verified as follows. Uniqueness of the perturbed periodic solution implies that all symmetries in $K \cap \Gamma_{\text{hexapod}-\text{robot}-2}$ fix the perturbed trajectory pointwise since the perturbed equations have $\Gamma_{\text{hexapod}-\text{robot}-2}$-equivariance, that is, they have $\Gamma_{\text{hexapod}-\text{robot}-2}$ symmetry group. So $K \cap \Gamma_{\text{hexapod}-\text{robot}-2} \subset K'$. Conversely, any symmetry in $K' \subset \Gamma_{\text{hexapod}-\text{robot}-2}$ must be in $K$ again by the uniqueness of hyperbolic periodic solutions in a small neighborhood. An analogous argument is valid for $H'$.

This raises some questions concerning applicability of these 'close' solutions to motor actuators in robots. How small should be $\|x(t) - \tilde{x}(t)\|$ so that these differences are forgotten by the physical mechanism? Considering consumer and educational robots, particularly considering prototype and research oriented devices, presently these apply "hobby "servos which include digital microcontrollers. Specifically, Dynamixel servos from Robotis, appear as an attractive and appealing type of servos, considering feasibility for precision control and dynamic operation. The capabilities of these servos, were extensively analyzed in [29]. Their tests have shown that the average uncalibrated servo has a position bias within $\pm 0.21^o$ at the ends of its range of motion, and that, when unloaded, the overall standard deviation of position feedback from target position is $0.36^o$. These tests have demonstrated that the servo accurately reaches its target position when unloaded.

Therefore, considering solutions that are close to the ideal solution, with an error smaller than $10^{-3}$ or $10^{-2}$, these considerations are not relevant for physical, real devices that employ these servos or alike.

## 3   Numerical simulations

We simulate the CPG model hexapod-robot. In each CPG-unit, the discrete part $y(t)$ is inserted as an offset of the rhythmic part $x(t)$. See Fig. 2 for a schematic diagram of the superposition of the two primitives.
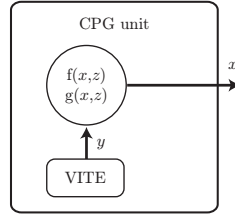


Figure 2: CPG unit of a CPG locomotor model for hexapod robots (hexapod-robot). Discrete primitive inserted as an offset.

We also consider three distinct couplings between the oscillators: diffusive, synaptic, and mixed.

We start from a stable periodic solution, purely rhythmic. Then, we vary parameter $T$ in steps of 0.1 in the interval $[0, 25]$. For each value of $T$, we perturb the rhythmic solution and simulate it until a stable periodic solution is obtained. Then we compute its amplitude and frequency values. We restart the simulations for a new value of $T$. Numerical results are illustrated.

The system of ordinary differential equations that models the discrete primitive is the VITE model given by [3]:

$$\begin{aligned} \dot{v} &= \delta(T - p - v) \\ \dot{p} &= G\,max(0, v) \end{aligned} \tag{3}$$

This set of differential equations generates a trajectory converging to the target position $T$, at a speed determined by the difference vector $T - p$, where $p$ models the muscle length, and $G$ is the go command. $\delta$ is a constant controlling the rate of convergence of the auxiliary variable $v$. This discrete primitive controls a synergy of muscles so that the limb moves to a desired end state, given a volitional target position.

The equations for each rhythmic motor primitive are known as the modified Hopf oscillator [15, 17, 4] and are given by:

$$\begin{aligned} \dot{x} &= \alpha(\mu - r^2)x - \omega z = f(x, z) \\ \dot{z} &= \alpha(\mu - r^2)z + \omega x = g(x, z) \end{aligned} \tag{4}$$

where $r^2 = x^2 + z^2$, $\sqrt{\mu}$ is the amplitude of the oscillation. For $\mu < 0$ the oscillator is at a stationary state, and for $\mu > 0$ the oscillator is at a limit cycle. At $\mu = 0$ it occurs a Hopf bifurcation. Parameter $\omega$ is the intrinsic frequency of the oscillator, $\alpha$ controls the speed of convergence to the limit cycle. $\omega_{swing}$ and $\omega_{stance}$ are the frequencies of the swing and stance phases, $\omega(z) = \frac{\omega_{stance}}{\exp(-az)+1} + \frac{\omega_{swing}}{\exp(az)+1}$ is the intrinsic frequency of the oscillator. With this ODE system, we can explicitly control the ascending and descending phases of the oscillations as well as their amplitudes, by just varying parameters $\omega_{stance}$, $\omega_{swing}$ and $\mu$.

The coupled systems of ODEs that model CPG hexapod-robot for synaptic, diffusive and mixed couplings are given by:

$$\begin{aligned} \dot{x}_i &= f_1(x_i, z_i, y_i) + k_1 h_1(x_{i+\varepsilon_i}, x_i) + k_2 h_2(x_{i-2}, x_i) \\ \dot{z}_i &= g_1(x_i, z_i, y_i) + k_3 h_1(z_{i+\varepsilon_i}, z_i) + k_4 h_2(z_{i-2}, z_i) \end{aligned} \tag{5}$$

where $f_1(x_i, z_i, y_i) = \alpha(\mu - r_i^2)x_i - \omega z_i$, $g(x_i, z_i, y_i) = \alpha(\mu - r_i^2)z + \omega x_i$, and $r_i^2 = (x_i - y_i)^2 + z_i^2$. $k_i$ are the coupling strengths. Indices are taken modulo 12 and $\varepsilon_i = (-1)^{i+1}$. The resulting system bifurcates between

a unique point attractor and a limit cycle according to one single parameter, $\mu$ (see below). It is believed that this design enables to produce more complex movements modeled as periodic movements around time varying offsets.

We simulate the CPG model (5). Parameter values used in the simulations are $\mu = 10.0$, $\alpha = 5$, $\omega_{\text{stance}} = 6.2832\,\text{rads}^{-1}$, $\omega_{\text{swing}} = 6.2832\,\text{rads}^{-1}$, $a = 50.0$, $G = 1.0$, $\delta = 10.0$. We considered three periodic solutions identified with three hexapod gaits, tripod, metachronal, and caterpillar. We computed amplitude and frequency values of the periodic solutions produced by CPG hexapod-robot and identified with these hexapod rhythms. We conclude that the amplitude and frequency values of the achieved stable periodic solutions, obtained after inserting the discrete to the rhythmic primitive, are not or are only slightly affected. Therefore, it is possible to use them for generating trajectories for the joint values of real robots, since varying the joint offset will not affect the required amplitude and frequency of the resultant trajectory, nor the gait.

## 4   Conclusion

We present a new CPG model for the locomotion rhythms of an hexapod robot, consisting of twelve CPG-units. We study the effect on two periodic solutions, produced by this CPG model hexapod-robot, identified with tripod, metachronal and caterpillar, of superimposing discrete and rhythmic primitives. We simulate the CPG model for synaptic and diffusive couplings. We compute the amplitude and the frequency values of the stable periodic solutions, obtained after inserting the discrete part into the rhythmic one, for values of the discrete primitive target parameter $T \in [0, 25]$. Numerical results show that amplitude and frequency values are almost constant for the two couplings. These results may be generalized for robots with a smaller or larger number of legs, since the CPG model used here easily extrapolates for $2n$ legs.

## Acknowledgments

## References

[1] P. Arena, L. Fortuna, M. Frasca, and L. Patané. CNN Based Central Pattern Generators with Sensory Feedback. *Proceedings of the 2002, 7th IEEE International Workshop* (2002).

[2] E. Bizzi, A d'Avella, P Saltiel and M Trensch. Modular organization of spinal motor systems. *The Neuroscientist* **8** No 5 (2002) 437–442.

[3] D. Bullock and S. Grossberg. *The VITE model: a neural command circuit for generating arm and articulator trajectories.* In J. Kelso, A. Mandell, and M. Shlesinger, editors, Dynamic patterns in complex systems, (1988) 206–305.

[4] R. Campos, V. Matos, C.P. Santos. Hexapod Locomotion: a Nonlinear Dynamical Systems Approach. *IECON 2010 - 36th Annual Conference on IEEE Industrial Electronics Society* , (2010) 1546–1551.

[5] J.J. Collins and I. Stewart. Hexapodal gaits and coupled nonlinear oscillators. *Biological Cybernetics* (1993) 287–298.

[6] S. Degallier and A. Ijspeert. Modeling discrete and rhythmic movements through motor primitives: a review. *Biological Cybernetics* **103** (2010) 319–338.

[7] S. Degallier, C.P. Santos, L. Righetti, and A. Ijspeert. Movement Generation using Dynamical Systems: A Drumming Hummanoid Robot. Humanoid's06 IEEE-RAS International Conference on Humanoid Robots. Genova, Italy (2006).

[8] M. Golubitsky and I. Stewart. *The symmetry perspective*, Birkhauser, (2002).

[9] M. Golubitsky, I. Stewart, P.-L. Buono, and J.J. Collins. A modular network for legged locomotion. **Physica D 115** (1998) 56–72.

[10] M. Golubitsky, I. Stewart, P.L. Buono, and J.J. Collins. Symmetry in locomotor central pattern generators and animal gaits, *Nature* **401** (1999) 693–695.

[11] S. Grillner, P. Wallén, K. Saitoh, A. Kozlov, B. Robertson. Neural bases of goal-directed locomotion in vertebrates - an overview. *Brain Research Reviews* **57** (2008) 2–12.

[12] M.A. Lewis, A.H. Fagg, and G.A. Bekey. Genetic Algorithms for Gait Synthesis in a Hexapod Robot. *Zheng, ed. Recent Trends in Mobile Robots, World Scientific*, New Jersey, (1994) 317–331.

[13] G.L. Liu, M.K. Habib, K. Watanabe, and K. Izumi. Central pattern generators based on Matsuoka oscillators for the locomotion of biped robots, *Artif Life Rootics* **12** (2008) 264–269.

[14] K. Maneesilp, B. Purahong, and P. Sooraksa. A New Analog Control Circuit Design for Hexapod using Cellular Neural Network. *The 30th Annual Conference of the IEEE Industrial Electronics Society*, November 2-6, Busan, Korea, (2004).

[15] J. Marsden, and M. McCracken. *Hopf Bifurcation and Its Applications.* New York: Springer-Verlag, (1976).

[16] V.A. Makarov, E. del Rio, M.G. Bedia, M.G. Velarde, W. Ebeling. Control pattern generator incorporating the actuator dynamics for a hexapod robot. *Int. J. Appl. Math. Comp. Sci* **3** No.2 (2007) 97–102.

[17] V. Matos, C.P. Santos, C.M.A. Pinto. A Brainstem-like Modulation Approach for Gait Transition in a Quadruped Robot. *Proceedings of The 2009 IEEE/RSJ International Conference on Intelligent RObots and Systems, IROS 2009* St Louis, MO, USA, October (2009).

[18] M. Oliveira, C.P. Santos, M. Ferreira. Simulated Visually-Guided Paw Placement During Quadruped Locomotion. In *Proceedings of the 35th Annual Conference of the IEEE Industrial Electronics Society*, Porto, Portugal (2009).

[19] K.G. Pearson. Common Principles of Motor Control in Vertebrates and Invertebrates, *Annual Review of Neuroscience* **16** (1993) 265–297.

[20] C.M.A. Pinto, C.P.Santos, D. Rocha, V. Matos. Impact of Discrete Corrections in a Modular Approach for Trajectory Generation in Quadruped Robots. *AIP Conf. Proc. NUMERICAL ANALYSIS AND APPLIED MATHEMATICS ICNAAM 2011: International Conference on Numerical Analysis and Applied Mathematics*, **1389** Issue 1 Halkidiki, Greece (2011) 509–513.

[21] C.M.A. Pinto, C.P.Santos, D. Rocha, V. Matos. A Modular Approach for Trajectory Generation in Biped Robots. *AIP Conf. Proc. NUMERICAL ANALYSIS AND APPLIED MATHEMATICS ICNAAM 2011: International Conference on Numerical Analysis and Applied Mathematics*, **1389** Issue 1 Halkidiki, Greece (2011) 495–499.

[22] C.M.A. Pinto, C.P.Santos, D. Rocha, V. Matos. A New CPG Model for the Generation of Modular Trajectories for Hexapod Robots. *AIP Conf. Proc. NUMERICAL ANALYSIS AND APPLIED MATHE-MATICS ICNAAM 2011: International Conference on Numerical Analysis and Applied Mathematics*, **1389** Issue 1 Halkidiki, Greece (2011) 504–508.

[23] L. Righetti and A.J. Ijspeert. Design methodologies for central pattern generators: an application to crawling humanoids. *Proceedinhs of Robotic Science and Systems* (2006) 191–198.

[24] C. Santos. Generating timed trajectories for an autonomous vehicle: a non-linear dynamical systems approach. *ICRA. IEEE INTERNATIONAL CONFERENCE ON ROBOTICS AND AUTOMATION*, New Orleans. (2004) ISBN 0-7803-8232-3.

[25] U. Saranli, M. Buehler, and D.E. Koditschek. Design, Modeling and Preliminary Control of a Compliant Hexapod Robot. *Proceedings of ICRA 2000* (2000) 2589–2596.

[26] U. Saranli, M. Buehler and D.E. Koditschek. Rhex - A Simple and Highly Mobile Hexapod Robot. *International Journal of Robotics Research* **20** (7) (2001) 616-631.

[27] G. Schöner, M. Dose. A dynamical systems approach to tasklevel system integration used to plan and control autonomous vehicle motion. *Robotics and Autonomous Systems* **10** (4) (1992) 253–267.

[28] G. Schöner and C. Santos. Control of movement time and sequential action through attractor dynamics: A simulation study demonstrating object interception and coordination. *Proceedings of the 9th Intelligent Symposium on Intelligent Robotic Systems - SIRS'2001*, Toulouse, France (2001).

[29] Ethan Tira-thompson. Digital Servo Calibration and Modeling. DOI: 10.1.1.187.7677

[30] S.X. Yang, M. Meng. An efficient neural network approach to dynamic robot motion planning. *Neural Networks* **13** 2 (2000) 143–148.